A reduced size continuation algorithm for rotating Schrodinger equation

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Abstract

The laser rotating Bose-Einstein Condensates is governed by the rotating Schrodinger equation. The dynamical formation of wave function is observed by the continuation method that solves the parameterized nonlinear equations discretized by spectral method. The proposed continuation algorithm takes the scatter length as parameter for a reduced system, followed by an algorithm taking angular velocity as parameter. The scheme is more efficient due to the size of the nonlinear equation being reduced in half.

Problem description

Bose-Einstein condensates(BEC) is a state of matter of a gas of bosons cooled to temperatures very close to absolute zero, that is, near 0 K. This state is described by the wave function $\Psi(\mathbf{z},t)$ in the nonlinear Schrodinger equation.

$$\begin{cases} i\Psi = -\frac{1}{2}\Delta\Psi + V(\mathbf{z})\Psi + \mu|\Psi|^2\Psi - \omega L_z\Psi, & t>0, \mathbf{z}\in\Omega\subseteq\mathbb{R}^2 \\ \Psi(\mathbf{z},t)=0, & t\geq 0, \mathbf{z}\in\partial\Omega \end{cases},$$

where $\boldsymbol{i}=\sqrt{-\mathbf{1}}$, $\mathbf{z}=(x,y)\in\mathbb{R}^2$, $\Psi(z,t):\mathbb{R}^2\times\mathbb{R}\to\mathbb{C}$ is the wave function, $\nabla=\partial_{xx}+\partial_{yy}$ is the Laplace operator, $\mathbf{V}(\mathbf{z})$ is an external trapping potential, $\boldsymbol{\mu}\in\mathbb{R}$ is the interact scattering length, $\boldsymbol{\omega}$ is the angular velocity, the operator $L_z=-i(x\partial y-y\partial x)$ is the z component of the angular momentum, and Ω is a bounded and smooth domain.

Algorithm

Separating real and imaginary parts, we obtain

$$\begin{cases} -\frac{1}{2}\Delta R(\mathbf{z}) + V(\mathbf{z})R(\mathbf{z}) + \mu|\phi(\mathbf{z})|^2R(\mathbf{z}) - \omega(xT_y - yT_x) = \lambda R(\mathbf{z}), \mathbf{z}\in\Omega \\ -\frac{1}{2}\Delta T(\mathbf{z}) + V(\mathbf{z})T(\mathbf{z}) + \mu|\phi(\mathbf{z})|^2T(\mathbf{z}) + \omega(xR_y - yR_x) = \lambda T(\mathbf{z}), \mathbf{z}\in\Omega \\ R(\mathbf{z}) = T(\mathbf{z}) = \mathbf{0}, \text{ on } \partial\Omega \end{cases}$$

Discretize the system into rotating nonlinear Schrodinger equation $\boldsymbol{F}:\mathbb{R}^{2N+2}\to\mathbb{R}^{2N+1}$

$$F(\mathbf{a},\mathbf{b},\lambda,\omega) = \begin{cases} F_1(\mathbf{a},\mathbf{b},\lambda,\omega) = [L+V-\lambda G]\mathbf{a} + \mu\mathbf{h} - \omega\mathbf{s} = \mathbf{0} \\ F_2(\mathbf{a},\mathbf{b},\lambda,\omega) = [L+V-\lambda G]\mathbf{b} + \mu\mathbf{k} + \omega\mathbf{t} = \mathbf{0} \\ e(\mathbf{a},\mathbf{b},\lambda,\omega) = h^2[\mathbf{c}\circ(G\mathbf{a})]^T(G\mathbf{a}) + h^2[\mathbf{c}\circ(G\mathbf{b})]^T(G\mathbf{b}) - 1 = 0 \end{cases}$$

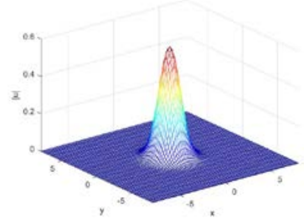
The following subsystem is solved by the continuation method with initial value given by linear Schrodinger equation.

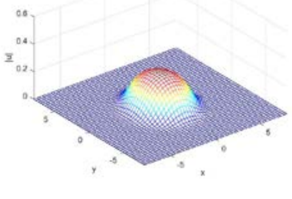
$$\hat{F}(\mathbf{a},\lambda,\mu) = \begin{cases} F_1(\mathbf{a},\lambda,\omega) = [L+V-\lambda G]\mathbf{a} + \mu\hat{\mathbf{h}} = \mathbf{0} \\ e(\mathbf{a},\lambda,\omega) = h^2[\mathbf{c}\circ(G\mathbf{a})]^T(G\mathbf{a}) - 1 = 0 \end{cases}$$

Once we have a solution $\hat{\boldsymbol{u}}$ of subsystem, then $(\hat{\boldsymbol{u}},\hat{\boldsymbol{u}})$ is a solution of equation $F(\mathbf{a},\mathbf{b},\lambda,0)=\mathbf{0}$

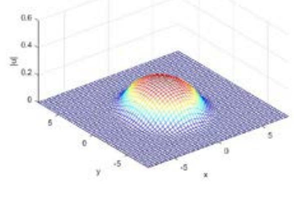
Results $\boldsymbol{\Omega}:[-8,8]^2$, $\mathbf{V}(\mathbf{z})=\frac{1}{2}(1.44x^2+y^2)$

Initial solution:

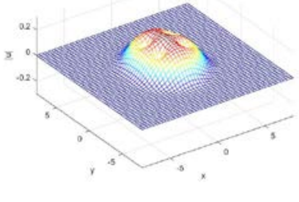




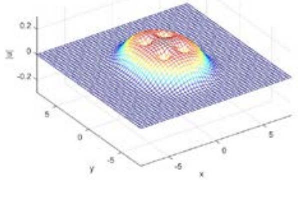
$\mu=100$、$\omega=0$



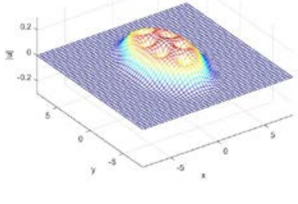
$\mu=200$、$\omega=0$



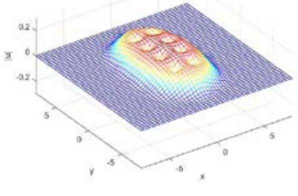
$\mu=100$、$\omega=0.68$



$\mu=200$、$\omega=0.66$



$\mu=100$、$\omega=0.88$



$\mu=200$、$\omega=0.86$

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