Macroscopic Traffic Simulation of Signalized Ring Road and its Effective Flow Rates

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Abstract

In this research, the interrupted flow problem of LWR model at signalized intersections is defined as conservation laws with discontinuous coefficients and then a Godunov-type numerical scheme is proposed to solve the problem. The proposed model is illustrated through numerical experiments under different traffic conditions and results are examined.

Contributions of this research include: to define the LWR model for interrupted flow conditions at signalized intersections and solve the problem with a revised Godunov scheme.

Problem description

The car density $u(x,t)$ in the closed ring road can be set as:

$$u_t + [B(x,t)f(u)]_x = 0, \quad t>0, \quad x\in[-R,R],$$

with boundary condition $u(-R,t) = u(R,t)$.

Where $f(u) = v_M u(1-\frac{u}{u_M})$, and

$$B(x,t) = \begin{cases} 1, & \text{otherwise,} \\ 0, & \text{if} \quad x=0, \quad t - jT_c \in [\pi T_c, T_c], \quad j = 0,1,2,\dots \end{cases}$$

$\pi \in [0,1]$ is the greenlight ratio, T_c is the cycle length time, and πT_c is the effective greenlight time.

Through computing the density and flow variation we have the cumulative flow and use it to apply the effective flow rate $\bar{f}$.

Then construct the Macroscopic fundamental diagram by collecting $\bar{f}$ under different u_0 varies from $[0, u_M]$.

Conclusions

A numerical scheme is proposed to solve the problem of conservation laws with discontinuous interface and the proposed scheme is then applied to solve the LWR model under interrupted flows, especially signals.

The proposed scheme is then illustrated and evaluated through different numerical experiments, and instead of TFD, a more complicated density-velocity relation, namely the Greenshield model, is used.

Discussion

In the redlight situation, we have to calculate the Riemann problem between $[U_{-\frac{1}{2}}, U_{\frac{1}{2}}]$ in two aspects, one is from the left where the information stops in front of the signal, and one the other one is from the right where we can not receive information after the signal. The left Riemann problem set as $H_{0^-} = H(U_{-\frac{1}{2}}, U_L^*)$ and the right Riemann problem set as $H_{0^+} = H(U_R^*, U_{\frac{1}{2}})$.

Greenlight scheme time $t - kT_c \in [0, \pi T_c]$, $k = 0,1,2,\dots$

$$U_{i+\frac{1}{2}}^{j+1} = U_{i+\frac{1}{2}}^{j} - \frac{\Delta t}{\Delta x}(F_{i+1}^j - F_i^j), \quad i = \dots,-2,-1,0,1,2,\dots$$

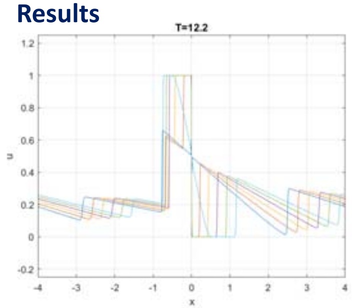
Redlight scheme time $t - kT_c \in (\pi T_c, T_c]$, $k = 0,1,2,\dots$

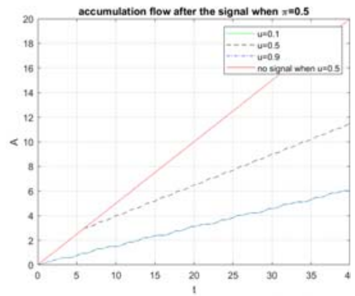
$$\begin{cases} U_{i+\frac{1}{2}}^{j+1} = U_{i+\frac{1}{2}}^{j} - \frac{\Delta t}{\Delta x}(F_{i+1}^j - F_i^j), & i \notin [-1,0], \\ U_{-\frac{1}{2}}^{j+1} = U_{-\frac{1}{2}}^{j} - \frac{\Delta t}{\Delta x}(H_{0^-}^j - F_{-1}^j), \\ U_{\frac{1}{2}}^{j+1} = U_{\frac{1}{2}}^{j} - \frac{\Delta t}{\Delta x}(F_1^j - H_{0^+}^j). \end{cases}$$

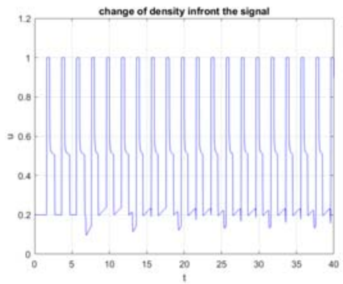
Where

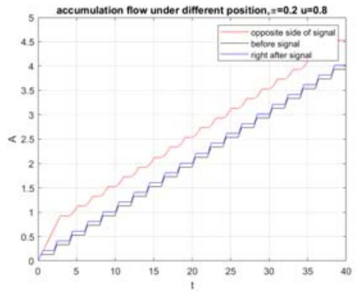
$$\begin{cases} F_i = \min(f(\min(U_{i-\frac{1}{2}}, u_{cf})), f(\max(u_{cf}, U_{i+\frac{1}{2}}))), \\ H_{0^-} = \min(f(\min(U_{-\frac{1}{2}}, u_{cf})), 0), \\ H_{0^+} = \min(0, f(\max(u_{cf}, U_{\frac{1}{2}}))). \end{cases}$$

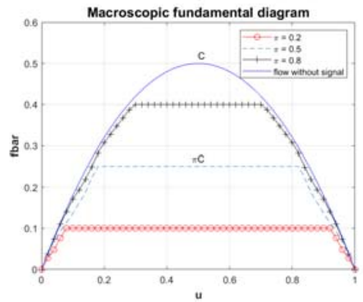
Results











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