

額外源點法在三維外域聲場虛擬頻率問題之數值分析

Shu-Wei Hsu (許書璋), Department of Civil Engineering, Tamkang University, Taiwan (606380383@s06.tku.edu.tw)
 Advisor: Prof. Jia-Wei Lee (李家璋)

Abstract

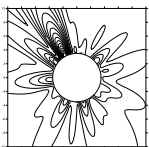
額外源點法可以填補在間接法中沒有內部零場方程式的不足，因此使用基本解法(MFS)即使是在虛擬頻率的情況也可求得二維外域聲場問題的準確解。然而此法目前只應用在二維的案例並且也有失效點的風險存在。基於以上原因，本論文將有兩個延伸方向，其一是將增加額外源點法推廣至三維外域聲場問題，其二是使用雙層勢能與混合勢能來取代額外源點的單層勢能。我們將採用退化核函數來表現閉合型的基本解，同時也將一併使用退化核解析推導其額外源點為單層勢能時可能失效點位置。為了驗證本想法的有效性，我們考慮了含球形輻射體的無限域聲場問題。在球形幾何的問題下我們考慮四個不同的案例，分別是單根與三重根且再分為無解與無窮多解的情況。

Discussion

在三重根的情況下，失效點的分析是三個點中，任兩點不能一樣。還有當三個點的 θ 固定且兩個點的 ϕ 一樣時，第三個點的 ϕ 不能與前面兩個點的 ϕ 一樣且不能夾180度。最後選得出在額外單層勢能的徑向失效點，在雙層勢能的情況下能來解決。

$$k=4.4934$$

$$n=1, m=1 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$



$$s_1(0.1388, 0.0048, 0.7878)$$

$$s_2(0.1389, 0.0024, 0.7878)$$

$$s_3(-0.1387, -0.0073, -0.7878)$$

額外源點在失效點

Problem description

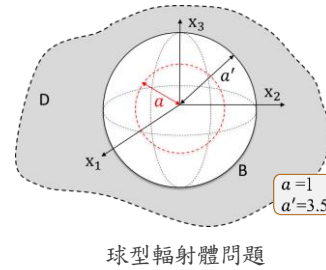
控制方程: $(\nabla^2 + k^2)u(\mathbf{x}) = 0$ 軟邊界條件(Dirichlet條件): $u(\mathbf{x}) = \bar{u}(\mathbf{x})$

傳統基本解法: $u(\mathbf{x}) = \sum_{j=1}^N \alpha_j U(s_j, \mathbf{x})$ 基本解: $U(\mathbf{s}, \mathbf{x}) = -\frac{e^{ikr}}{4\pi r}, r = |\mathbf{x} - \mathbf{s}|$

額外雙層勢能: $u(\mathbf{x}) = \sum_{i=1}^N \gamma_i U(s_i, \mathbf{x}) + \sum_{j=1}^{Nr} \beta_j^D \frac{\partial U(s_j, \mathbf{x})}{\partial n(s_j)}, s_i \in B', s_j \notin B'$

額外單層勢能: $u(\mathbf{x}) = \sum_{i=1}^N \gamma_i U(s_i, \mathbf{x}) + \sum_{j=1}^{Nr} \beta_j U(s_j, \mathbf{x}), s_i \in B', s_j \notin B'$

額外混合勢能: $u(\mathbf{x}) = \sum_{i=1}^N \gamma_i U(s_i, \mathbf{x}) + \sum_{j=1}^{Nr} \beta_j^M \left(U(s_j, \mathbf{x}) + \kappa \frac{\partial U(s_j, \mathbf{x})}{\partial n(s_j)} \right), s_i \in B', s_j \notin B'$

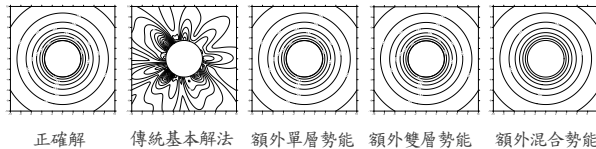


球形輻射體問題

Results

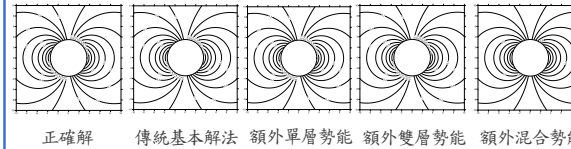
$k=3.1416$ $j_0(ka) = 0$ **Case 1**

$$n=0, m=0 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$



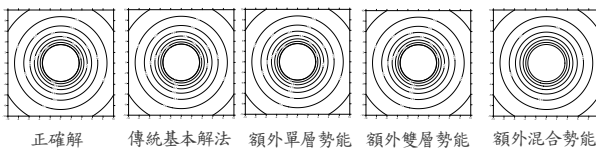
Case 2 $j_0(ka) = 0$ $k=3.1416$

$$n=1, m=1 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$



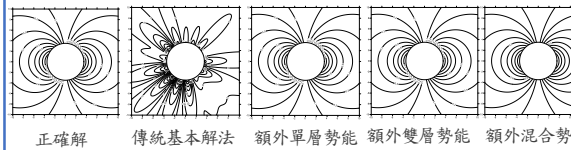
$k=4.4934$ $j_1(ka) = 0$ **Case 3**

$$n=0, m=0 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$



Case 4 $j_1(ka) = 0$ $k=4.4934$

$$n=1, m=1 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$

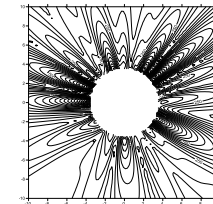


Conclusions

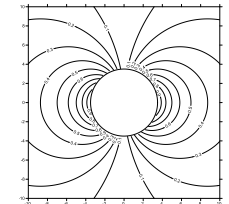
- ◆ 成功的將額外源點法推展到三維
- ◆ 失效點的分析可以正確地避免失效的位置
- ◆ 額外單層勢能、額外雙層勢能與額外混合勢能都可以解決虛擬頻率的問題。

$$k=3.1416$$

$$n=1, m=1 \quad u(x) = \frac{h_n^{(1)}(k\rho_x)}{h_n^{(1)}(ka)} P_n^m(\cos\theta_x) \cos(m\phi_x)$$



額外單層勢能



額外雙層勢能

References

1. J. W. Lee, C. F. Nien, and J. T. Chen, "Combination of the CHIEF and the self-regularization technique for solving 2D exterior Helmholtz equations with fictitious frequencies in the indirect BEM and MFS," in Symposium of the International Association for Boundary Element Methods (IABEM 2018), Paris, France (2018).
2. J. W. Lee, J. T. Chen, and C. F. Nien, "Indirect boundary element method combining extra fundamental solutions for solving exterior acoustic problems with fictitious frequencies," J. Acoust. Soc. Am. 145(5), 3116-3132 (2019).